$\square$ Code No.: 12006 AS
VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD
B.E. (CBCS) II-Semester Advanced Supplementary Examinations, June/July-2017

## Engineering Mathematics-II

Time: $\mathbf{3}$ hours
Note: Answer ALL questions in Part-A and any FIVE from Part-B

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\text { Part-A }(10 \times 2=20 \text { Marks })
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1. What is meant by rank of a matrix? Give an example of a $4 \times 3$ matrix whose rank is 2 .
2. Find the index, signature of the quadratic form $x_{1}^{2}+2 x_{2}^{2}-3 x_{3}^{2}$
3. Solve $x d y+2 y d x=x y d y$.
4. Find the general solution of the differential equation $y^{\prime}+2 x y=x e^{-x^{2}}$.
5. Solve $y^{\prime \prime}+3 y^{\prime}-4 y=0$, if one of its solution is $y_{1}=e^{x}$.
6. Find a particular integral of $y^{\prime \prime}+6 y^{\prime}+9 y=\sin 2 x$.
7. Express $x^{3}+2 x^{2}+x-1$ in terms of Legendre polynomials $P_{n}(x)$.
8. Find the value of $P_{n}^{\prime}(-1)$.
9. Compute $\Gamma(1 / 2)$.
10. Find the general solution of the differential equation $x^{2} y^{\prime \prime}+x y^{\prime}+\left(x^{2}-\frac{1}{16}\right) y=0$ in terms of Bessel functions.

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\text { Part-B }(5 \times 10=50 \text { Marks })
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(All bits carry equal marks)
11. a) Determine whether the vectors $(1,-1,0),(0,1,-1),(0,2,1),(1,0,3)$ are linearly dependent. If so, find the relation between them.
b) Diagonalize the matrix $A=\left(\begin{array}{ccc}3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2\end{array}\right)$.
12. a) Solve $\left(3 x^{2} y^{4}+2 x y\right) d x+\left(2 x^{3} y^{3}-x^{2}\right) d y=0$.
b) A generator having e.m.f. 100 volts is connected in series with a 10 ohms resistor and an inductor of 2 henries. If the switch is closed at a time $t=0$, find the current at time $t>0$.
13. a) Apply the method of variation of parameters to solve $y^{\prime \prime}-2 y^{\prime}=e^{x} \sin x$.
b) Find the general solution of Euler-Cauchy equation $x^{2} y^{\prime \prime}-2 x y^{\prime}-4 y=6 x^{2}+4 \log x$.
14. a) State the generating function for Legendre polynomials and hence prove that
i) $P_{n}(1)=1$
ii) $P_{n}(-1)=(-1)^{n}$ and
iii) $P_{n}(-x)=(-1)^{n} P_{n}(x)$.
b) Prove that $\int_{-1}^{1} P_{m}(x) P_{n}(x) d x=0$, if $m \neq n$.
15. a) State and prove the relation between Beta and Gamma functions.
b) Express $J_{4}(x)$ in terms of $J_{0}(x)$ and $J_{1}(x)$.
16. a). Verify Cayley-Hamilton theorem for $A=\left[\begin{array}{lll}3 & 2 & 4 \\ 4 & 3 & 2 \\ 2 & 4 & 3\end{array}\right]$ and hence find $A^{-1}$.
b) Find the orthogonal trajectories of the family of curves $r=c(1+\cos \theta)$, where $c$ is a parameter.
17. Answer any two of the following:
a) Find all non zero solutions of the boundary value problem

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y^{i v}-k^{4} y=0, y(0)=0, y^{\prime \prime}(0)=0, y(\pi)=0, y^{n}(\pi)=0
$$

b) Find the power series solution of the differential equation $\left(1-x^{2}\right) y^{\prime \prime}-2 x y^{\prime}+2 y=0$ about the origin.
c) Evaluate $\int_{0}^{\infty} \frac{x^{a}}{a^{x}} d x, a>1$, using Beta and Gamma functions.

